A Stiffness Criterion For Cooling Time Estimation

Haoyu Xu and David Kazmer*

Department of Mechanical and Industrial Engineering

University of Massachusetts Amherst

Amherst, Massachusetts 01003

* Author to whom correspondence should be addressed.

Prof. David Kazmer
Eng. Lab. Bldg.
Amherst, MA 01003
(413) 545-0670
Fax (413) 545-1027
kazmer@ecs.umass.edu

Abstract

The cooling time of an injection molded part directly affects the production cost and efficiency. In order to accurately estimate the cooling time, it is necessary to define the ejection condition. One common criterion for part ejection is when the mean or maximal temperature is equal or less than an estimated ejection temperature, T_e [1]. But in industrial practice, the ejection condition is determined by the part's deformation or warpage due to ejection forces and subsequent cooling [2]. This research uses a combination of experimental, analytical, and statistical methods to discuss different ejection criteria and their rationality, thereby developing a more effective criterion based on part stiffness. This stiffness criterion facilitates more accurate estimation of cooling time at early stages of product development. This knowledge will help the designer to improve the design and reduce the cost by increasing the production efficiency while simultaneously ensuring injection molded part quality.

Introduction

Injection molding is a cyclic process of forming plastic into a desired shape by forcing the molten polymeric resin under pressure into an evacuated cavity. The plastic melt solidifies inside the shaped cavity, and then is ejected from the cavity after a given period of time, the cooling time [3]. Cooling time typically occupies more than one third of the whole molding cycle. As such, cooling time offers the largest opportunity for cycle time reduction and increased production rates. In addition, the quality of the final product, production efficiency, and manufacturing cost are significantly affected by the cooling stage. Therefore, estimating and optimizing cooling time play an important role in manufacturing operations. Although cooling time is dependent on many factors, such as resin properties, cooling system and processing conditions, the cooling time estimate is based on the assumed ejection condition [4].

The most common ejection condition is that the center-line or maximal temperature of the part is equal to or less than T_e, the ejection temperature [5]. Ballman and Shusman [6] presented a cooling time model to predict the cooling time of a given injection molded part:

$$CT = \frac{h^2}{2\pi\alpha}\ln\left[\frac{\pi}{4}\left(\frac{T_i - T_m}{T_e - T_m}\right)\right] \tag{1}$$

where CT is the cooling time in seconds; h is maximum cavity thickness, cm; α is the thermal diffusivity, cm^2/s; T_i is the injection temperature, ºC; T_m is the mold temperature, ºC; T_e is the ejection temperature, ºC. This equation was developed using the following assumptions:

- The material is fully solidified and rigid below T_e;
- The geometry of the molding will not exert a major influence on the temperature at which it will "freeze";
- Die filling is isothermal or nearly so;
- The temperature of the melt entering the die cavity is known or can be reasonably derived from the press cylinder temperature;
- The surface of the die cavity remains at, or nearly at the die-cooling water temperature and is reasonably uniform;
- Heat transfer at the mold surface is assumed to be infinite and the effect of separation of the part and die surface due to shrinkage – even of thick sections preceded by small gates – can be ignored;
- The material exhibits constant thermal properties.

Utilizing these assumptions, Equation 1 will predict the cooling time such that the molded part will be ejected when the innermost temperature of its thickness section has reached the heat distortion temperature of the polymer.

Kwon and Opolski [7] required average temperature of the given part to be equal or less than the ejection temperature, T_e as the criterion to estimate the cooling time by:

$$CT = \frac{4h^2}{\pi^2 \alpha} \ln\left[\frac{8}{\pi^2}\left(\frac{T_i - T_m}{T_e - T_m}\right)\right] \tag{2}$$

Because the average temperature is used rather than the maximal, the cooling time will always be shorter than that predicted from equation 1. However, this model would not account for differences in part size and number of cavities, which may impact the heat loading on the cooling system. As such, Bush, Field and Rosato [5] used both theoretical and statistical methods to derive the following equation to give a good estimation of cooling time:

$$CT = 1.35\frac{h^2}{2\pi\alpha}\ln\left[\frac{8}{\pi^2}\left(\frac{T_i - T_m}{T_e - T_m}\right)\right] + 0.0151W_p N_{cav} + 8.87 \tag{3}$$

where W_p is part weight (g) and N_{cav} is the number of cavities in the mold. While such a semi-empirical approach provides more accurate time estimation for parts representative of those used in the study, the resulting method may not be of much value beyond the investigated area. For instance, Euqation 3 can not be used in the growing area of thin wall molding, in which cooling times are frequently less than 8.87 seconds.

Yu and Sunderland [1] similarly developed relations to estimate the cooling time of a given part by experimental molding of many different material systems, thicknesses, and process conditions:

$$CT = 5.23 - 26.7\frac{h^2}{\pi\alpha}\ln\frac{\pi\theta_e}{4} - 0.374\ln\theta_e, \tag{4}$$

$$CT = -0.064 - 27\frac{h^2}{\pi\alpha}\ln\frac{\pi\theta_e}{4} - 0.23\ln\theta_e, \text{ and} \tag{5}$$

$$CT = -8.24 - 38.3\frac{h^2}{\pi\alpha}\ln\frac{\pi\theta_e}{4} - 7.18\ln\theta_e, \tag{6}$$

where $\theta_e = \frac{T_e - T_m}{T_i - T_m}$. Equation 4 is for amorphous materials with thermal diffusivities greater than or equal to 10^{-3} cm^2/s and equation 5 is for amorphous materials with thermal diffusivities lower than 10^{-3} cm^2/s. Equation 6 is for semicrystalline materials.

Although Yu and Sunderland’s results agreed with the field data well, there are significant limitations. They have performed many experiments for different materials and utilized significant theoretical and statistical analysis, but the derived equations only apply to the specific part dimension, material and process conditions inspected in this study.

In industrial practice, parts are ejected at the highest possible temperature that does not present unacceptable sacrifices in quality. Four primary defects related to cooling time are presented in Table 1. The process engineer will adjust the cooling time and/or other molding parameters until the molded part meets the quality specification. Often there is a significant trade-off between part quality and cycle time. As such, the design and process engineer must rely on available models and expertise to optimize the design and process. Recently, commercial computer aided engineering softwares have been developed to relax many of the previous assumptions, including:

- Constant mold wall temperature and related contact conditions;
- Constant thermal properties;
- Isothermal initial conditions;
- 1-D heat path.

These simulations have been well validated and integrated in many new product development processes [3]. However, all of these approaches utilize T_e as the basis for ejection temperature. The exact specification of T_e is not well defined. Frequently the ejection temperature is set to some value less than the glass transition temperature – typically 10 ºC under T_g. Alternatively, the ejection temperature can be characterized by the heat deflection temperature which is not significantly more accurate. Thus, estimation of the solidification temperature severely limits the confidence in cooling time estimates.

Model Development

To develop better predictive capability, experimental, analytical, and statistical means are used to develop and validate different ejection criteria. The numerical treatment of heat transfer will first be developed. Afterwards, the concept of part stiffness as a criterion for ejection will be explicitly defined and validated.

Heat Transfer

The typical heat path in the cooling stage of injection molding is that heat is conducted from hot polymer to the comparatively cold mold, then conducted through the mold to the cooling line, where it is convected away by the coolant. The instant energy balance equation for the cooling stage of a given control volume of the part is [8],

$$-\frac{\partial q_x}{\partial x}dx-\frac{\partial q_y}{\partial y}dy-\frac{\partial q_z}{\partial z}dz+\dot{q}dxdydz=\rho c_p\frac{\partial T}{\partial t}dxdydz\,. \tag{7}$$

Assuming one dimensional cooling for illustrative purposes, the governing equation for instantaneous heat transfer for a controlled volume in the part and mold is,

$$\frac{kA}{\Delta x}(T_{i+1}{}^{p+1}-T_i{}^{p+1})+\frac{kA}{\Delta x}(T_{i-1}{}^{p+1}-T_i{}^{p+1})=\rho c_p A\frac{\Delta x}{2}\frac{T_i{}^{p+1}-T_i{}^{p}}{\Delta t}\,. \tag{8}$$

At the interface between the mold and the coolant, the heat transfer is given by,

$$hA(T_\infty{}^{p+1}-T_i{}^{p+1})+\frac{k_m A}{\Delta x}(T_{i\pm1}{}^{p+1}-T_i{}^{p+1})=\rho c_{pm}A\frac{\Delta x}{2}\frac{T_i{}^{p+1}-T_i{}^{p}}{\Delta t}\,. \tag{9}$$

At the interface between the mold steel and polymer melt, the heat transfer can be represented as,

$$
\begin{aligned}
&2F_pT_{i-1}{}^{p+1} + [1 + 2F_p + \frac{2\Delta x_p}{F_p}\frac{k_m}{k_p}\frac{F_m}{2\Delta x_m}(1 + 2F_p)]T_i{}^{p+1} + 2F_p\Delta x_p\frac{k_m}{k_p}\frac{1}{\Delta x_m}T_{i+1}{}^{p+1} \\
&= (1 + \frac{2\Delta x_p}{F_m}\frac{k_m}{k_p}\frac{F_p}{2\Delta x_m})T_i{}^p,
\end{aligned}
\tag{10}
$$

where $F_m = (\alpha_m\Delta t)/\Delta x_m{}^2$ for the mold and $F_p = (\alpha_p\Delta t)/\Delta x_p{}^2$ for the polymer.

Structural Integrity

Since the correct ejection temperature is not always known during application development, an alternative method for cooling time estimation is being proposed based on representative part stiffness. For all polymeric materials, the elastic modulus varies significantly with temperature as shown in Figure1 for a grade of ABS. Such information is widely available from material suppliers.

As the polymer melt cools in the mold, the solidified layers increase in stiffness while continuing to propagate towards the core. Our premise is that ejection criteria based on temperature are misleading and inaccurate, and that an ejection criterion based on part stiffness would more closely approximate industrial practice. Thus, a molded part, if sufficiently stiff, could be ejected regardless of the core or average temperature [9].

The relation between the bending curvature, bending load, and stiffness is,

$$
\frac{1}{c} \propto \frac{M}{IE} \quad . \tag{11}
$$

Equation 11 shows the inverse relation between bending curvature and stiffness. With increment of stiffness I, the curvature and permanent deformation will decrease. Stiffness was also chosen as the ejection criterion since it is a single aggregate value, derived from the elastic modulus that varies as a function of thickenss.

A fourth order divided-difference polynomial interpolation [10] was utilized to develop the temperature dependence of elastic modulus as shown in Figure 1. The dynamic part stiffness can be evaluated by integrating the elastic modulus with the second order of inertia,

$$I(t) = \int_0^h E(T(z,t))z^2 dz \ . \tag{12}$$

The stiffness criterion will exhibit an asymptotic behavior as the frozen layers propagate towards the core and approach the coolant temperature. This asymptotic can be computed as:

$$I(\infty) = E(T_{coolant}) \times h^3 / 3 \, . \tag{13}$$

Thus, the stiffness can be evaluated in both absolute and relevant terms. The results of several different ejection criteria for cooling time estimation will now be discussed.

Results

The solution of the previous heat equations, Equations 8 through 10, were obtained utilizing an implicit numerical method with twenty four layers in the melt region and ten layers for the mold region. Cooling simulations were performed for a flat plaque molded of ABS with sample thicknesses of 0.5, 1, 2, 3 and 4 mm. The processing conditions for the simulation are shown in Table 3, and represented typical molding operation. The thermal properties utilized in the study are shown in Table 2.

The dynamic temperature profile is shown in Figure 2 for a plaque of 4mm thickness. Initially the two mold halves are 50 ºC, with the melt at a uniform temperature of 250 ºC. Each of the inner curves shows the temperature change at an increment of 3 seconds. After 3 seconds, for instance, the core temperature has dropped only 2 ºC to 248 ºC while the mold wall has increased from 50 to 100 ºC. After 39 seconds, the core has dropped to 110 ºC while the mold wall has been cooled to 65 ºC.

Similar analyses are performed in commercial molding simulations, though with different material, design and processing conditions. The remainder of this section will now utilizing such cooling results to discuss the effectiveness of different ejection criteria.

Criterion 1: max (T) < T_e

As previously discussed, a common criterion for part ejection is when the maximal temperature of the polymer melt drops below some defined glass transition temperature or ejection temperature. For the 4mm thick plaque plotted in Figure 2, the centerline temperature drops below the ejection temperature of 110 ºC after 39 seconds. Such analyses can be repeated for different wall thicknesses, material properties, and process conditions for cooling time estimation. As the thickness increases, the cooling time increases due to: 1) the increased amount of heat which must be conducted to the coolant, and 2) the reduced heat conductance through the thicker section of insular plastic.

Requiring the maximal part temperature to fall below T_e is a conservative criterion, since it guarantees all temperatures in the part are less than T_e. While a conservative criterion provides a “safe” basis for costing, it will tend to provide an over-estimate of processing costs. Of course, the accuracy of cooling time estimate will be dependent upon accurate setting of T_e.

Criterion 2: mean (T) < T_e

A second commonly used criterion is that the mean part temperature be less than the ejection temperature. This is an aggressive assumption, since it does not guarantee that a large percentage of the part will have ejection temperatures less than T_e. Fortunately, the roughly parabolic shape of the temperature profiles will cause about 2/3 of the part to be less than T_e when this criterion is satisfied. For the plaque behavior plotted in Figure 2, the required cooling time is estimated to be 27 seconds, a significant reduction from the previous estimate of 39 seconds.

Figure 3 plots the estimated part cooling times for criteria 1 and 2 as a function of wall thickness. At small wall thicknesses, the two estimates converge as the mean and maximal part temperatures are almost equal. The difference between mean and maximal melt temperature at ejection increases from 2 ºC at 0.5mm to 22 ºC at 4mm. Thus, the two criteria diverge, driving a 10-second difference in cooling time estimates at ejection. Such a deviation in cooling time prediction is critical in industrial application – a 10-second error in cycle time represents a 20% loss in productivity and significantly greater loss in profitability. For the development team, it is unclear which estimation should be utilized. As such, they would be forced to choose criterion 1 or cooling time in accordance predictions with their previous experience.

Criterion 3: Part Stiffness = 73%

The estimated part stiffness can also be used as an ejection criterion. The part stiffness is calculated using Equations 12 and 13 with the material data of Figure 1 and temperature distribution of Figure 2. The resulting elastic modulus behavior is shown in Figure 4 for the 4mm thick plaque at several different thickness locations. Note how the modulus quickly approaches the nominal value at the skin, but rises more slowly at the core of the part.

The propagation of increased modulus from the skin to the core will lead to asymptotic behavior in the stiffness of the part. To normalize the calculated stiffness, we divide the dynamic stiffness by the stiffness for material at a uniform mold temperature, i.e.:

$$\tilde{I}(t) = \frac{I(t)}{I(t = \infty \Rightarrow T(z) = T_{mold})} . \tag{14}$$

The resulting normalized stiffness as a function of time is shown in Figure 5. For parts with thin walls, the stiffness increases very quickly to the asymptotic value. As the wall thickness increases, the stiffness increases more slowly, indicating that a larger cooling time is required to obtain the same normalized value.

To assess a representative value of stiffness, a wall thickness of 3.0 mm was selected from industry cooling data for ABS. From this data, a cooling time is 18 seconds was specified in practice. Using Figure 5, it was observed that this cooling time corresponded to a stiffness of 73%. This normalized value of stiffness was then assumed as the ejection cirterion for all wall thicknesses. The normalized stiffness can be calculated for several different wall thicknesses from industry data, as plotted as shown in Figure 6.

This intermediate result validates the stiffness criterion for the ejection condition. It is observed that for a significant range of different wall thicknesses, the normalized stiffness varies by only a few percent. This indicates that utilizing a normalized stiffness of 73% should provide good estimates of cooling time.

The stiffness criterion also indicates the behavior of the other two criteria. With reference to criterion 1 (max (T) < T_e), it should be noted that since the frozen layers propagated from the walls to the center, and that moment of inertia increases away from the neutral axis, that thicker parts may potentially be ejected while the core remains at temperatures higher than T_e. With reference to criterion 2 (mean (T) < T_e), the larger deviation between max and mean temperatures provide a part that is less stiff at larger wall thicknesses.

Figure 7 plots the estimated cooling time for all three ejection criteria as well as the cooling times from industry data. The correlation between industry data and predictions utilizing the stiffness criterion are very good. It should be noted, moreover, that adjusting the ejection temperatures for the other two criteria will result in a shift in the curve without any significant improvement in the global behavior.

Conclusion

This research has introduced a new ejection criterion for injection molding. Utilizing part stiffness as an ejection criterion has been shown to provide more accurate estimation of cooling time than temperature based criteria. This stiffness criterion has two significant advantages (beyond accuracy) than other ejection criteria. First, it requires no ejection temperature of the user, though now modulus data must be supplied. However, such data is frequently available from material suppliers.

Second, the part stiffness criterion can be used for process improvement. Given a specific quality attribute, such as warpage, a process engineer can inspect stiffness as a function of time and infer what added stiffness may be achieved by increasing the ejection time, or reducing the melt or mold coolant temperature. However, the effect of such process, material or part design change was not investigated in this paper and further research is required to assess the effectiveness of the stiffness criterion in industry application.

Nomenclature

h = convection heat transfer coefficient, W/m^2.K

ρ = density, kg/m^3.

k = thermal conductivity, W/m.K.

$\dot{q}$ = rate of energy generation per unit volume, W/m^3.

c_p = specific heat, J/kg.K.

A = area normal to heat transfer direction, m^2.

T = temperature, °C.

I = stiffness, Gpa m^3.

E = Youngs modulus, Gpa.

M = bending load, kN.m.

q = heat transfer rate, W.

c = radius, m.

dx = differential unit in x direction

dy = differential unit in y direction

dz = differential unit in z direction

Δx = controlled unit in x direction

Δy = controlled unit in y direction

Δz = controlled unit in z direction

Δt = controlled time interval

Subscript:

∞ = surrounding

i = node number

m = mold.

p = part.

inst = instant.

max = maximal.

mean = mean.

normal = normal.

percen = percent.

x = flow direction

y = cross flow direction

z = thickness direction

Superscript:

p = time step

Greek:

α = thermal diffusivity, m2/s.

σ = stress, N/m2.

μ = strain, m.

Reference

1. Yu, C.J. and J.E. Sunderland, *Determination of ejection temperature and cooling time in injection molding.* Polymer engineering and science, 1992. **32**(3): p. 191.
2. Rosato, D.V. and D.V. Rosato, eds. *Injection molding handbook*. 1985, Van nostrand reinhold company, New York.
3. AC Technology, *C-Mold design guide*. 1995, Ithaca, New York: AC Technology.
4. Moller, J., *et al.*, *Post-ejection cooling behavior of injection molded part.* ANTEC' 96, 1996.
5. V., B.J., *et al.*, *Computer aided part cost analysis for injection molding.* RETEC' 88, 1988(October).
6. Ballman, R. and T. Shusman, *Easy way to calculate injection molding set-up time.* Modern Plastics, 1959(November): p. 126.
7. Tang, L.Q., *et al.*, *Three-dimensional transient mold cooling analysis based on Galerkin finite element formulation with a matrix-free conjugate gradient technique.* International Journal for Numerical Methods in Engineering, 1996. **39**(18): p. 3049-3064.
8. Chen, S.C. and Y.C. Chung, *Numerical simulations of the cyclic, transient mold heat transfer in injection model cooling process.* International Communications in Heat and Mass Transfer, 1994. **21**(3): p. 323-332.
9. Walsh, S.F., *Shrinkage and warpage prediction for injection molded components.* Journal of Reinforced Plastics and Composites, 1993. **12**(7): p. 769-777.
10. Cha, S. and F.S. Lai. *Analytical study of thin-wall injection molding: Filling and cooling analysis*. in *Annual Technical Conference - ANTEC, Conference Proceedings*. 1996. Indianapolis In USA.
11. Dowler, B. and L. Coppari, *How low can you go? Optimizing the cooling of small injection molded parts.* Plastics Engineering, 1997. **53**(6): p. 29-32.
12. V., S. and Patankar, *Numerical heat transfer and fluid flow*. 1989, New York: Hemisphere publishing corporation.
13. Hanselman, D. and B. Littlefield, *The student edition of matlab*. 1995, Englewood Cliffs: Prentice hall.
14. Gerald, G.F. and P.O. Wheatley, *Applied numerical analysis*. 5 ed. 1994, New York: Addison-Wesley publishing company.

List of Tables

Table 1: Defects related to cooling time

Table 2: Properties of part & mold

Table 3: Processing condition

List of Figures

Figure 1: Youngs modulus of ABS (Cycolac GPM 5500)

Figure 2: Temperature profile, criterion: Max(T) < T_e

Figure 3: Cooling times comparison of criteria 1 & 2

Figure 4: Youngs modulus variation with part thickness ant time

Figure 5: Dynamic stiffness ratio variation for different part thickness

Figure 6: Stiffness ratio comparison of different criteria & experiment data

Figure 7: Cooling times comparison of different ejection criteria & experiment data

Table 1: Defects related to cooling time

Defect	Description
Push-pin	While forcing part out of mold, ejector pins transmit excessive force to non-solidified wall sections, thereby extruding protrusions on surface of part
Shrinkage	Specific volume of polymer varies from the processing temperature to the ambient temperature
Warpage	Pliable part experiences out of plane deformation due to ejection forces or non-uniform cooling upon ejection
Residual stress	Differential shrinkage or non-uniform cooling. High molding pressures not resolved during part solidification

Table 2: Properties of part & mold

	Part	Mold
Material type	Cycolac GPM 5500	P-20
Density ρ (kg/m^3)	1005	7820
Thermal conductivity (W/m.K)	0.22	36.6
Specific heat C_p (J/kg.K)	2352.4	461.2

Table 3: Processing condition

Initial melt temperature (ºC)	250
Initial Mold temperature (ºC)	50
Coolant temperature (ºC)	50
Coolant thermal convection coefficient (W/m^2.K)	1000
Material ejection Temperature (ºC)	110

Figure 1: Youngs modulus of ABS (Cycolac GPM 5500)

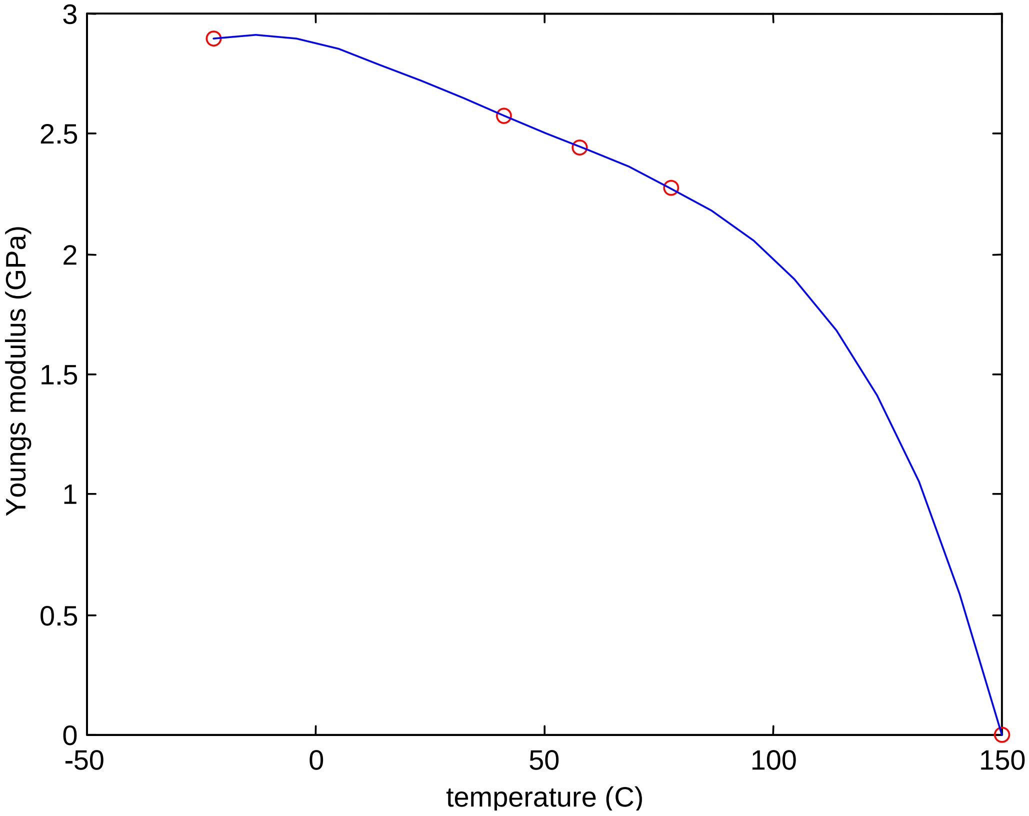

Figure 2: Temperature profile, criterion: Max(T) < T_e

Part thickness = 4mm

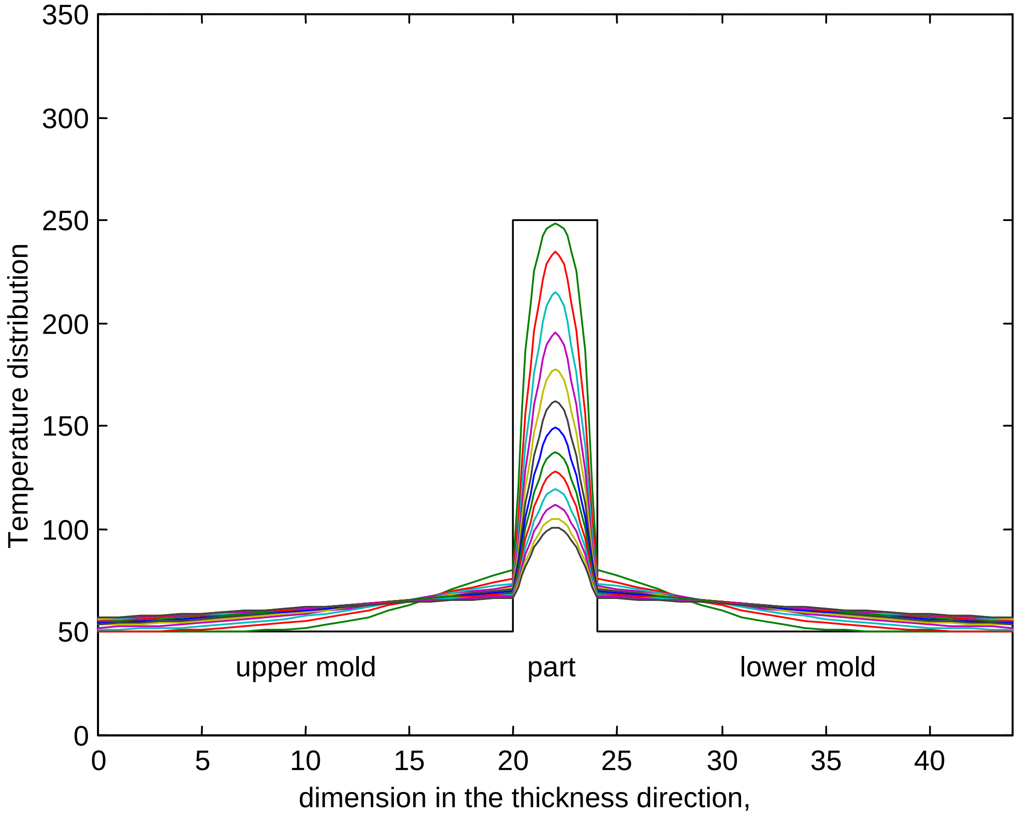

Figure 3: Cooling times comparison of criteria 1 & 2

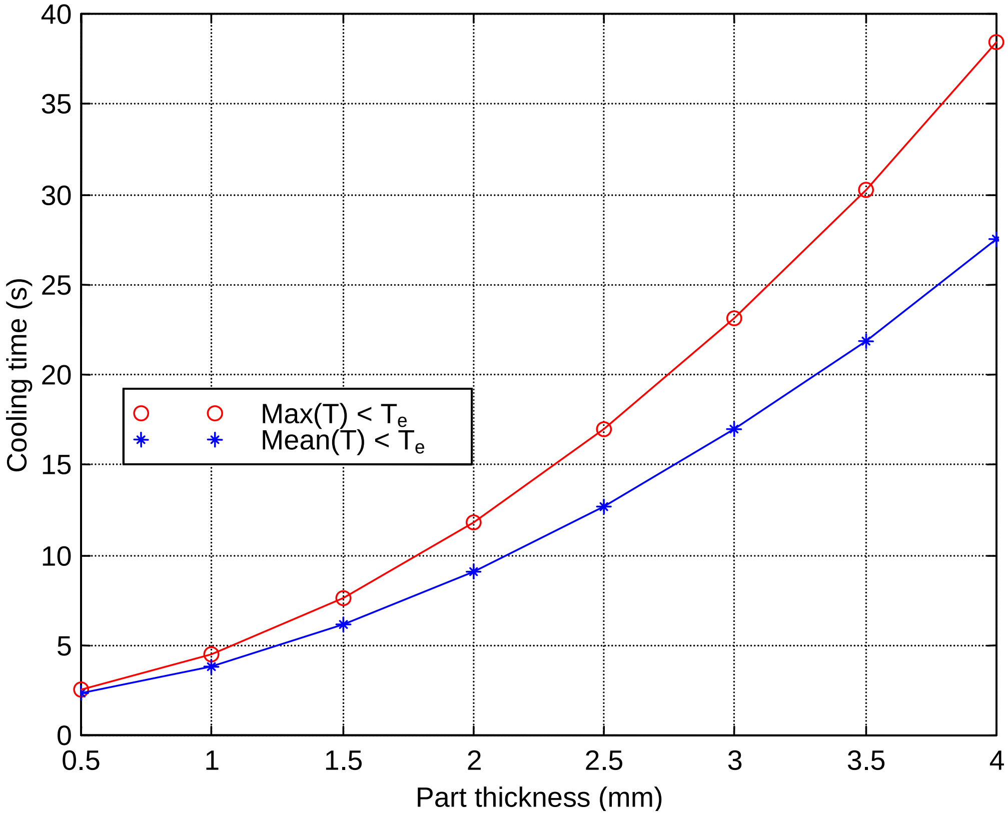

Figure 4: Youngs modulus variation with part thickness ant time

Part thickness = 4mm

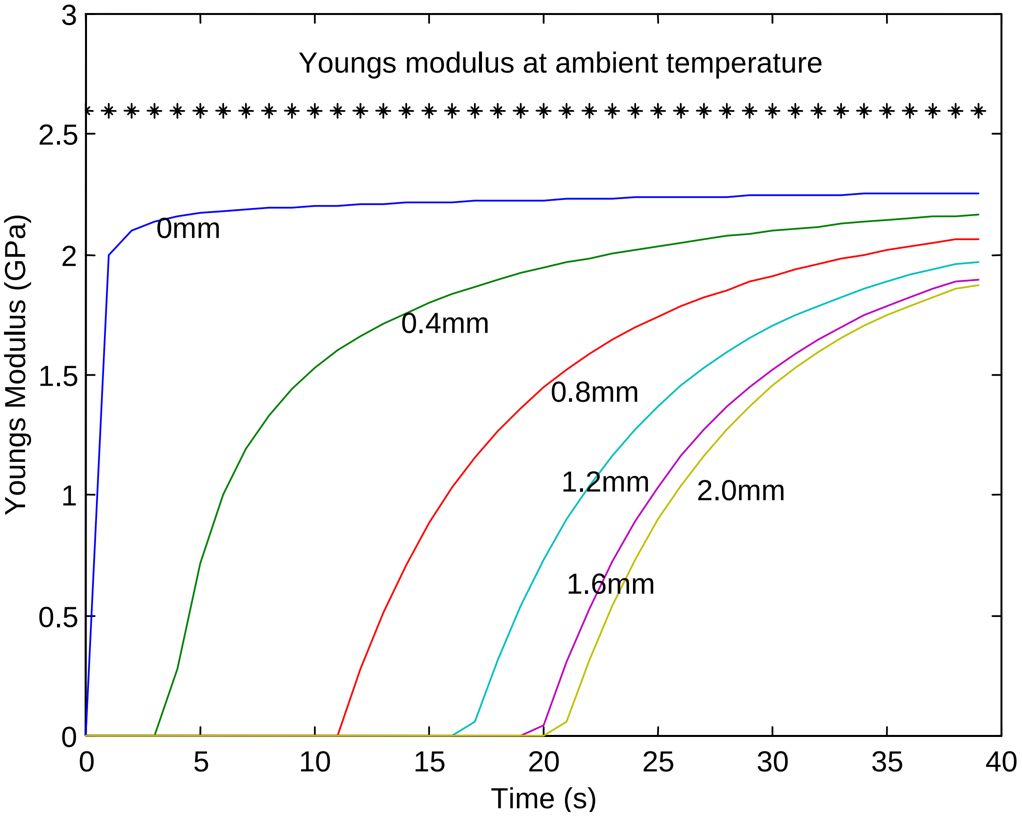

Figure 5: Dynamic stiffness ratio variation for different part thickness

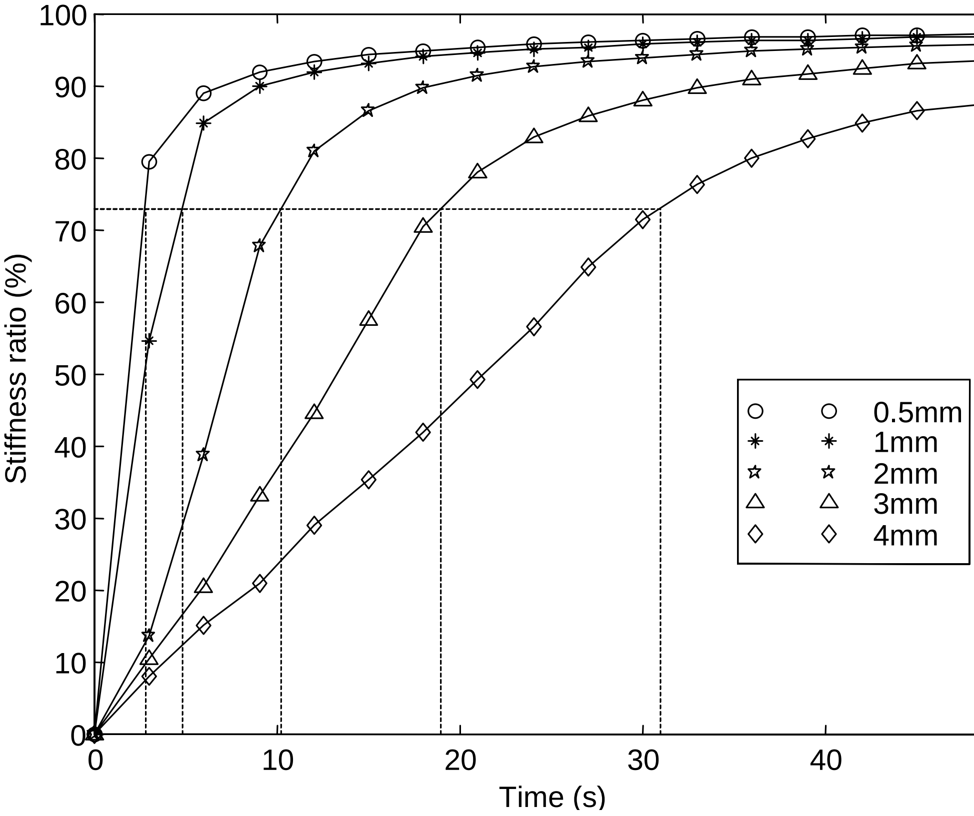

Figure 6: Stiffness ratio comparison of different criteria & experiment data

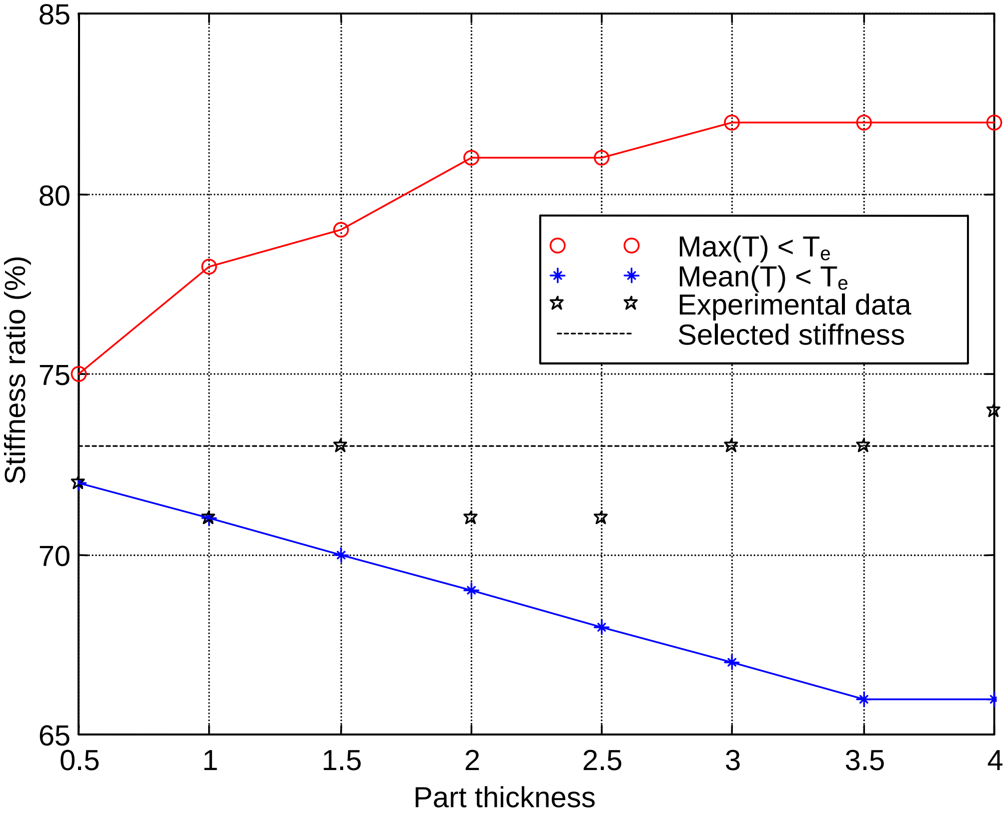

Figure 7: Cooling times comparison of different ejection criteria & experiment data