PREDICTION OF PRODUCTION YIELDS IN INJECTION MOLDING I

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Abstract

Plastics molders need to continuously improve production efficiency to remain competitive. With increasingly tight specifications driven by Six Sigma quality initiatives, however, such efficiency gains are more difficult to maintain. This paper investigates the use of process capability indices for linear and non-linear regression models based on observed shrinkage data for polypropylene molding parts. Statistical validation of the yield estimates is accomplished through Monte-Carlo analysis. The discrepancy among results indicates that current practices in industry are poor, and care should be taken when using yield prediction methods for process optimization or development.

Introduction

"Faced with the need to raise productivity to survive, especially against low-cost competitors, in such nations as China, more companies are pushing towards so-called lights-out manufacturing. Once a science-fiction dream, the phenomenon is emerging in plants and factories throughout the U.S. as machines become more reliable in making flawless parts on their own." [1]

There can be little doubt that such lights-out plastics processing must be more heavily utilized, and perhaps become the norm, if the Americas are to maintain a significant manufacturing industry. Consider the desired performance of an "exemplary" lights-out facility with ten molding machines running twenty-four hours per day with labor available only during the first shift. If each machine is molding four cavities per mold with thirty second average cycle time, then approximately 75,000 parts will be automatically produced on the second and third shifts.

Given the state of molding technology, it is not realistically feasible to run open loop (without quality control systems) in such a lights-out operation. As such, process sensors and quality estimators need to be developed to detect and discard defects from good molded samples. If such automated quality control systems are to be used successfully, then not only must the nominal quality of the moldings should be very high, but the assurance level of the quality controller must be very high as well. The optimal, integrated design of the molding machine and the quality controller is a function of the cost of accepting defective moldings and discarding acceptable moldings as well as the cost of sensors, system development, and validation.

In the foregoing example, assume that all the molding machines are instrumented with quality controllers that discard 98% of all molded defects. If the molder's customer is practicing Six Sigma guidelines, they may mandate no more than three defects per million purchased components. Even with the quality controllers, the molding machines must still produce a yield of 99.985% good moldings. Such a high production yield requires not only good mold design and excellent machine capability, but also exceptional process optimization and validation.

Analysis

Consider the distribution of a quality attribute between a lower specification limit, *LSL*, and an upper specification limit, *USL*, in Figure 1. The yield is defined as the probability of producing acceptable moldings between the required specification limits:

$$yield = P(LSL < y < USL) \tag{1}$$

or 100% minus the probability of violating either the lower or upper specification limit:

$$yield = 1 - P(y < LSL) - P(y > USL) \tag{2}$$

From Figure 1, it is clear that the process is not centered and violates the *USL* more frequently than the LSL. However, an excessive amount of defects would likely be produced even if the process were centered. As such, the variation in the quality attribute needs to be reduced through process optimization or development, or the specification limits must be broadened.

In industry, the process capability index, *Cp*, is defined as a ratio between the breadth of the specifications to the observed standard deviation:

$$Cp = \frac{USL - LSL}{6\sigma} \tag{3}$$

wherein a process capability of one indicates six deviations between the specifications corresponding to a possible yield of 99.87% [2]. As indicated in Figure 1, however, a non-centered process will result in a greater number of defects even though *Cp* remains unchanged. As such, the asymmetric process capability index, *Cpk*, is frequently utilized:

$$Cpk = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right) \tag{4}$$

since it measures the relative distance from the mean, μ, to the closest specification limit.

It is important to note that both the mean, μ, and the standard deviation, σ, are selected not random or arbitrary. Specifically, it is customary to set up the process such that the mean and standard deviation provide an acceptable production yield. With regards to this process optimization, it is common to utilize a regression model derived from a design of experiments [3] since it is not economical to statistically sample the global process space through a trial and error approach [4]. A second order response model with interactions is often used to quantify the quality, y, as a function of the process settings, x_j:

$$y = b_0 + \sum b_j x_j + \sum \sum b_i b_j x_{ij} \quad (5)$$

Once a valid response model is developed, it is also possible to estimate the standard deviation. Specifically, the moment matching method states that the variance on y is driven by the sum of the products of the variance on x with the sensitivity between y and x:

$$\sigma_y = \sqrt{\sum_{j=1}^{m} \frac{dy}{dx_j} \sigma_{x_j}^2} \quad (6)$$

where m is the number of process variables. Equation (6) is often used since it is expensive to derive a good estimate for the standard deviation of quality throughout the design space, and yet it is desirable to select a robust set of process conditions x that minimize σ_y, thereby maximizing the yield.

Experimental

Shrinkage is one of several important factors that determine the acceptance of molded parts. In this study, a commercial polypropylene grade (MFI=3.5 g/10min) was injection molded in a rectangular cavity with dimensions 150mm in length, 80mm in width and 3.2mm in thickness. The laminar gate was 1.2 mm thick (Fig1). The experimental trials were carried on in an Auburg 270V injection molding machine, with 80 ton clamp force. Crystalline polymers exhibits higher shrinkage than amorphous, as the packing is higher in crystalline regions

In order to evaluate the influence of process conditions on shrinkage of the injection molded plaques, four process variables were chosen: mold temperature (x_1), melt temperature (x_2), injection rate (x_3) and hold pressure (x_4). These variables were reported in literature as the most important in shrinkage analysis. Holding time and cooling time were kept constant for all the experimental conditions. The range of process settings was defined by the machine and material limits. The minimum and maximum melt temperatures were 200 and 260C, respectively. The mold temperature limits were 28 and 90C, as the coolant liquid was water. The injection rate was changed from 10 to 105 cc/s. The maximum holding pressure was set to 60 MPa, based on the clamp force and projected area of the part. The minimum holding pressure chosen, 20 MPa, provided moldings without sink marks.

The design of experiments and associated shrinkage data are provided in Table 1, in which the settings for different x_j are normalized and the resulting shrinkage varies from a minimum of 1.18% to 2.04% [5].

Results & Discussion

In the fitting of a response model to the experimental data, many computer programs default to, and many practioners utilize, full second or higher order models that consume all degrees of freedom available from experimental data. In other words, the model coefficients are chosen such that the resulting models perfectly match the data but provide meaningless behaviors between the data points [6].

To investigate the prediction capability of different response models with increasing number of model coefficients, a progressive set of optimal models were developed to minimize the average absolute error, *AAE*, between the predicted and observed shrinkage values. The resulting models are provided in Table 2, in which a zero indicates that the model term is not significant. For instance, the first row of Table 2 indicates that if the model only has one term, then it should be the intercept, y_0, equal to the average shrinkage of 1.47% and a resulting *AAE* of 10.276%. As additional coefficients and terms are added, the *AAE* drops substantially. Figure 2 plots the *AAE* as a function of number of model coefficients. It is observed that the shrinkage can be modeled with just a few coefficients. As such, a model with seven coefficients was selected:

$$\begin{aligned} y = 1.469 + 0.1279x_1 - 0.008x_2 - 0.144x_4 \\ +0.0294x_1^2 + 0.009x_2^2 + 0.0174x_1x_2 \end{aligned} \quad (7)$$

The plot of predicted shrinkage compared to the observed shrinkage shown in Figure 3 verifies the validity of the model across the processing space. Since the model is normalized, it is possible to make some interesting observations regarding the shrinkage behavior. First, the shrinkage is not significantly dependent on the injection rate, x_3. Second, the mold temperature and hold pressure are the most significant process settings, with behaviors as expected and consistent with the literature [7]. Third, this result indicates that the variances of the mold temperature and hold pressure are most significant in determining the shrinkage variance. Fourth, the three second-order terms indicate that the standard deviation is a significant function of the mold and melt temperatures.

It is desired to select a processing point that will maximize the number of acceptable moldings that meet the dimensional requirements. In this application, an *LSL* of 1.55% and a *USL* of 1.65% were used to correspond to a tight (but not excessive) tolerance requirement. To select an optimal processing point, a Monte Carlo analysis [8, 9] was utilized to exhaustively quantify the yield throughout the processing space and select the point with the maximum yield. Briefly, a Monte Carlo analysis generates a large set of statistically distributed processing conditions about a specified setpoint, then evaluates the regression model to identify whether the shrinkage was within the specification limits. By simulating many runs about the setpoint, the Monte Carlo analysis can provide an accurate yield estimate for complex response surfaces and arbitary statistical distributions.

The resulting plot of yield as a function of mold and melt temperature is shown in Figure 4. It is observed that there is only a small band of operating conditions that will produce acceptable moldings with yields exceeding 80%, corresponding to mold temperatures in the range of [45, 55C] and melt temperatures in the range of [215,230C].

Outside this range, the yield degrades quickly to the point where no acceptable moldings should be expected. Observing Figure 4, a process engineer might select nominal mold and melt temperatures of 50C and 220C, respectively, as indicated by the shaded circle in the figure.

It is now possible to compare the yield estimates resulting from standard industry usage of *Cp* and *Cpk* indices with those of the Monte Carlo analysis. The Excel functions utilized in estimating the deviations, means, and probabilities are as follows:

σ=T41=(ABS(C41*var1)+ABS(D41*var2)+ABS(E41*var3)+ABS(F41*var4)+ABS(2*G41*dim1*var1)+ABS(2*H41*dim2*var2)+ABS(2*I41*dim3*var3)+ABS(2*J41*dim4*var4)+ABS(K41*dim1*var2)+ABS(K41*var1*dim2)+ABS(L41*dim1*var3)+ABS(L41*var1*dim3)+ABS(M41*dim1*var4)+ABS(M41*var1*dim4)+ABS(N41*dim2*var3)+ABS(N41*var2*dim3)+ABS(O41*dim2*var4)+ABS(O41*var2*dim4)+ABS(P41*dim3*var4)+ABS(P41*var3*dim4))^0.5 (8)

μ=U41=Q41+C41*dim1+D41*dim2+E41*dim3+F41*dim4+G41*dim1^2+H41*dim2^2+I41*dim3^2+J41*dim4^2+K41*dim1*dim2+L41*dim1*dim3+M41*dim1*dim4+N41*dim2*dim3+O41*dim2*dim4+P41*dim3*dim4 (9)

$p(y>LSL)$=V41=NORMSDIST((1.55-U41)/T41) (10)

$p(y>USL)$=W41=NORMSDIST((1.65-U41)/T41) (11)

yield=W41-V41 (12)

where dim_j indicates the normalized value of x_j and var_j indicates the standard deviation of x_j and the column letters for other cells correspond to the columns of Table 2 such that cell C41 corresponds to the cell for the model coefficient on x_1.

To estimate the standard deviation on the shrinkage, it is necessary to know the variance of the machine process conditions. Hunkar [10] has characterized and described a machine evaluation methodology that quantifies the process consistency of any molding machine, which categorizes machines into capability classes from 1 to 9 with pre-defined variances as shown in Table 3. In general, it is anticipated that a higher class machine with reduced ranges will produce more consistent, higher precision molded parts. The values in Table 3 provide the standard deviations on the processing conditions x_j to estimate the yield for low and high class factor machines.

The results are in Figure 5, which compares the yield prediction accuracy of linear and non-linear responses surface models as well as the symmetric process capability index *Cp* to the asymmetric process capability index *Cpk* for different class factor machines. There are many interesting results. First, the Monte Carlo analysis estimated yields of 81% and 100% for the low and high class factor machines. These values represent the best yield estimates that can be obtained for this data, and indicate that upgrading from a class 1 machine is probably desirable in this application. However, it may be possible that a machine lower than class 9 may more economically produce a very high yield. Such class factor optimization is not the main focus of this paper.

All other set of bars correspond to current industry practices and indicate significant errors for yield prediction. In general, the use of the process capability indices for yield prediction grossly under-estimated the yields predicted with Monte Carlo analysis. However, this specific tendency of under-estimating the yield in this application should not be generalized since it is specific to the response surface models and process variations utilized. Indeed, long run manufacturing studies indicate that processes typically exhibit significantly broader statistical tails than characterized by a short run variation study [11]. Rather, it is hoped that these results instruct that the explicit and focused use of the process capability index may lead to acceptance of defective products, or rejection of acceptable products.

Other insights can be gained with respect to linear vs. non-linear models, and the use of *Cp* vs *Cpk*. Specifically, these results confirm that yield predictions based on the asymmetric process capability index *Cpk* will always be lower than those based on *Cp* (compare 95.6% to 94.5% and 96.6% to 89.7%). Further, these results suggest that the non-linear model will be more accurate since it considers the dependence of the standard deviation on the process conditions. However, the specific impact of non-linearity in the response surface model on yield can not be generalized (compare the increase from 95.6% to 96.6% with the decrease from 94.5% to 89.7%).

Conclusions

High production yields require not only good mold design and excellent machine capability, but also exceptional process optimization and validation. Design of experiments, response surface methods, and valid yield estimation techniques must be used in concert to develop and optimize competitive molding processes.

The current estimates of yield based on the process capability indices are very poor, and can really only be used in a qualitative fashion. A singular focus on the process capability index as an estimate of yield is misguided, and may lead to acceptance of defective products or rejection of acceptable products.

Yield estimation is vital to process improvement. General improvements in Hunkar class factors are not necessarily efficient. Rather, the process development team must consider how to get “the most bang for the buck.” In the example provided in this paper, improving the consistency of the injection rate would have minimal effect on the yield, and investment should focus on improving the mold temperature.

This paper focused on yield for one single quality attribute. Yield estimation and optimization of multiple quality attributes is much more difficult and involves significant issues related to information theory, combinatorics, and decision theory. The ultimate goal is to develop a process window algorithm that considers input variation and model uncertainty for multiple process conditions and quality attributes. The ideal system, envisioned in Figure 6, would start with a qualitative or unvalidated model and efficiently explore, model, and map the global feasible space until a true optimal processing point is found.

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Table 1: Shrinkage DOE Factors & Responses

Run	Condition	Mold	Melt	Vel	Pack	Shrinkage
1	38/215/34/30	-1	-1	-1	-1	1.64
2	72/215/34/30	1	-1	-1	-1	1.77
3	38/245/34/30	-1	1	-1	-1	1.47
4	72/245/34/30	1	1	-1	-1	1.76
5	38/215/82/30	-1	-1	1	-1	1.53
6	72/215/82/30	1	-1	1	-1	1.77
7	38/245/82/30	-1	1	1	-1	1.44
8	72/245/82/30	1	1	1	-1	1.76
9	38/215/34/50	-1	-1	-1	1	1.26
10	72/215/34/50	1	-1	-1	1	1.47
11	38/245/34/50	-1	1	-1	1	1.21
12	72/245/34/50	1	1	-1	1	1.47
13	38/215/82/50	-1	-1	1	1	1.26
14	72/215/82/50	1	-1	1	1	1.47
15	38/245/82/50	-1	1	1	1	1.25
16	72/245/82/50	1	1	1	1	1.51
17	55/230/58/40	0	0	0	0	1.47
18	28/230/58/40	-1.54	0	0	0	1.27
19	90/230/58/40	2	0	0	0	1.85
20	55/200/58/40	0	-2	0	0	1.52
21	55/260/58/40	0	2	0	0	1.49
22	55/230/10/40	0	0	-2	0	1.45
23	55/230/105/40	0	0	2	0	1.47
24	55/230/58/20	0	0	0	-2	1.77
25	55/230/58/60	0	0	0	2	1.18
26	45/220/40/45	-0.57	-0.67	-0.75	0.5	1.38
27	60/240/40/20	0.29	0.67	-0.75	-2	1.81
28	30/210/20/25	-1.43	-1.83	-1.58	-1.5	1.65
29	28/230/58/30	-1.54	0	0	-1	1.5
30	80/220/90/20	1.43	-0.67	1.37	-2	2.04

Table 3: Allowable Variation by Hunkar Class Factor

Parameter	Low (Class 9)	High (Class 1)
Mold Temperature (C)	8	2
Melt Temperature (C)	5	1
Injection Rate (cc/sec)	0.17	0.04
Holding Pressure (Mpa)	0.5	0.1

Table 2: Successively Defined Regression Models with Corresponding Average Absolute Error

#Coeffs	x1	x2	x3	x4	x1*x1	x2*x2	x3*x3	x4*x4	x1*x2	x1*x3	x1*x4	x2*x3	x2*x4	x3*x4	y0	AAE
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.470	10.276
2	0	0	0	-0.15	0	0	0	0	0	0	0	0	0	0	1.470	6.683
3	0.1258	0	0	-0.147	0	0	0	0	0	0	0	0	0	0	1.491	2.202
4	0.12	0	0	-0.146	0.0308	0	0	0	0	0	0	0	0	0	1.473	1.759
5	0.1288	0	0	-0.145	0.0329	0	0	0	0.025	0	0	0	0	0	1.476	1.559
6	0.1252	-0.012	0	-0.149	0.0302	0	0	0	0.0189	0	0	0	0	0	1.473	1.363
7	0.1279	-0.008	0	-0.144	0.0294	0.009	0	0	0.0174	0	0	0	0	0	1.469	1.266
8	0.1253	-0.008	0	-0.147	0.0302	0.0076	-0.005	0	0.0172	0	0	0	0	0	1.474	1.227
9	0.1241	-0.011	0	-0.145	0.0306	0.0063	-0.006	0	0.0191	0	0	0	0.0049	0	1.473	1.203
10	0.1128	-0.015	0	-0.14	0.0298	0.0049	-0.005	0	0.0201	0	0.0123	0	0.0099	0	1.470	1.132
11	0.112	-0.015	0	-0.14	0.0312	0.0049	-0.005	-0.001	0.02	0	0.013	0	0.01	0	1.470	1.130
12	0.112	-0.015	0	-0.14	0.0312	0.0049	-0.005	-0.001	0.02	0	0.013	0	0.01	0	1.470	1.130
13	0.1121	-0.014	0	-0.14	0.0278	0.0047	-0.006	0.001	0.0148	0.0084	0.0143	0	0.0038	0.0089	1.474	1.096
14	0.1113	-0.008	0	-0.146	0.023	0.0056	-0.008	-0.001	0.0135	0.0069	0.014	0.009	0.0051	0.0023	1.483	1.082
15	0.1052	-0.007	0.005	-0.143	0.0273	0.0087	-0.003	0.0023	0.0125	0.0019	0.0142	0.0081	0.0031	0.0049	1.470	1.030

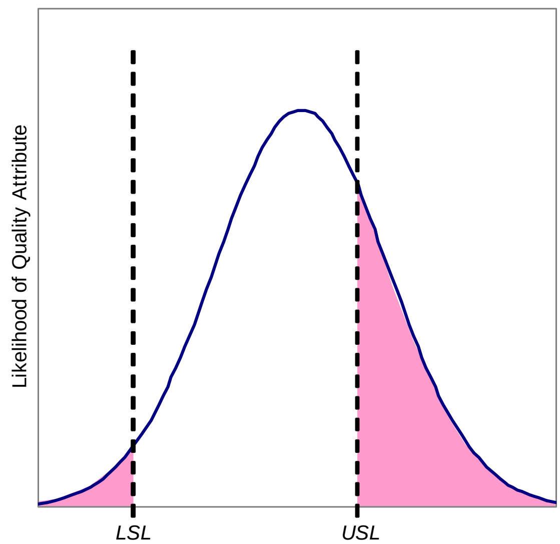

Figure 1: Distribution of Quality Attributes for Estimation of Yield

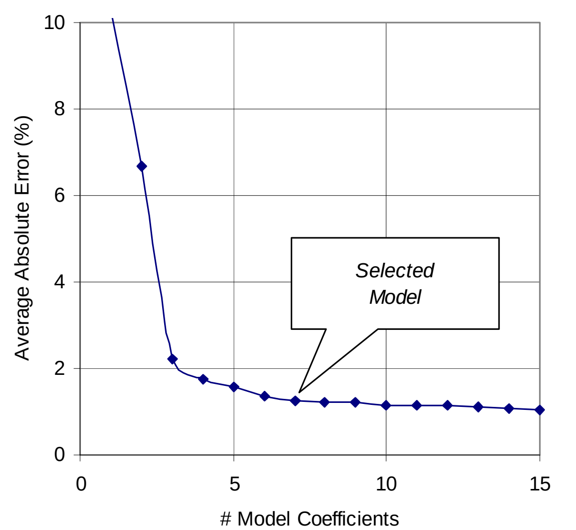

Figure 2: Selection of Number of Model Coefficients

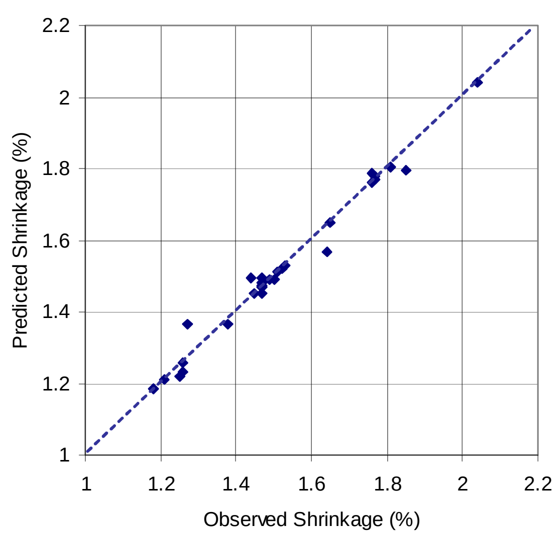

Figure 3: Verification of Statistical Model

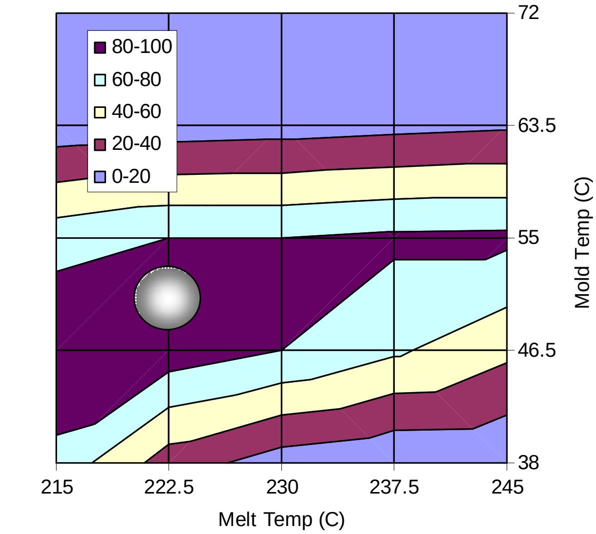

Figure 4: Yield Estimation With Monte Carlo Method to Identify Optimal Processing Point

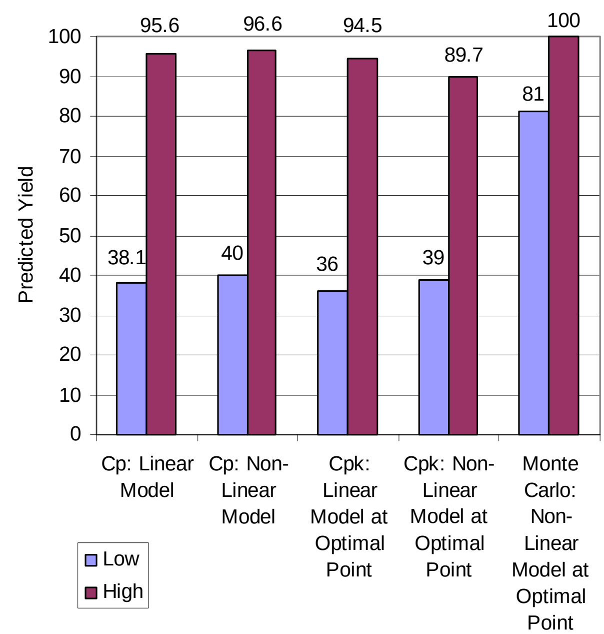

Figure 5: Comparison of Yield Estimation Techniques

Figure 6: Iterative Exploration & Optimization